RELIABILITY, MAINTAINABILITY & AVAILABILITY INTRODUCTION

International Society of Logistics (SOLE)
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System Reliability

- **The probability of performing a mission action without a mission failure within a specified mission time t**
- **A system with a 90% reliability has a 90% probability that the system will operate the mission duration without a critical failure**
- **The failure rate, Lambda, provides the frequency of failure occurrences over time**
- **The random variable in Reliability is time-to-failure**

$$\frac{1}{\lambda} = MTTF$$ **(Mean Time To Failure)**

- **The Reliability equation for a system has the failure rate times the mission time distributed exponentially, Reliability *R*(*t*) is given by:**

$$R(t) = e^{-\lambda t}$$ **(λ** = failure rate)

Additional Time to Failure Terminology

- **Mean Time Between Operational Mission Failure (MTBOMF) – System mission reliability often associated to an operating mission requirement, where the failure causes a mission abort or mission degradation**
- **Mean Time Between Failure (MTBF) – System reliability typically associated to a design specification based on operating use. Per failure definition, the failure may be to any item causing a logistics demand or just critical items within the system**
- **Mean Calendar Time Between Failure (MCTBF) – System reliability typically associated to a system operational availability based on calendar time per failure**
- **Failure Factor (FF) – Component logistics reliability typically used for logistics support expressed in terms of failures or demands per 100 systems per year**

System Requirement Example

- What is the MTBOMF of a system required to have a 91% reliability over a 72 hour mission pulse?

$$R(t) = e^{-\lambda t} = e^{\frac{-t}{MTBOMF}}$$

$$.91 = e^{\frac{-72}{MTBOMF}} \qquad \ln(.91) = \ln\left[e^{\frac{-72}{MTBOMF}}\right]$$

$$-0.0943107 = \frac{-72}{MTBOMF} \qquad 0.0943107 \times MTBOMF = 72$$

$$MTBOMF = \frac{72}{0.0943107} = 763.4$$ operating hours per mission failure

System Reliability Terminology

- **System - Collection of components, subsystems and/or assemblies arranged to a specific design in order to achieve desired functions with acceptable performance and reliability**
  - **The types of components, their quantities, their qualities and the manner in which they are arranged within the system have a direct effect on the system's reliability**
  - **The reliability relationship between a system and its components is sometimes misunderstood or oversimplified**
    - **An example non-valid statement is: *If all components in a system have a 90% reliability at a given time, the reliability of the system is 90% for that time.***

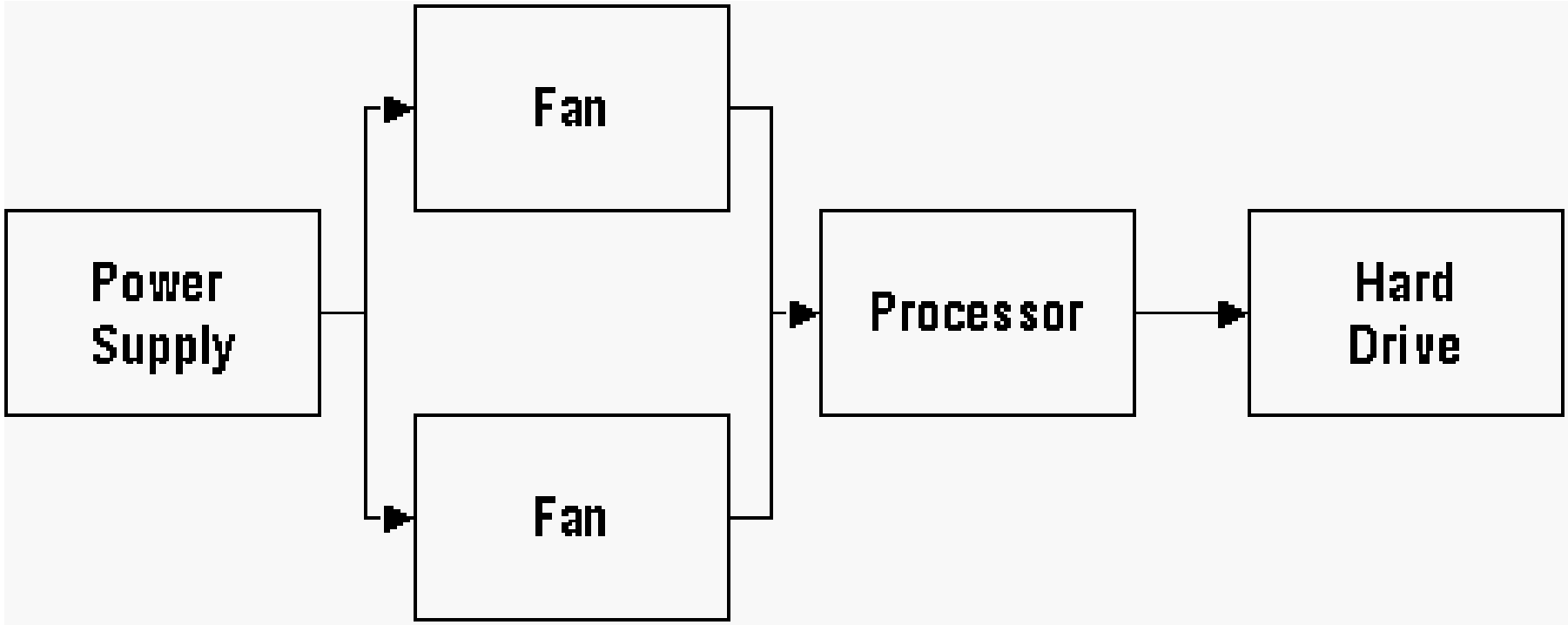
System Reliability Terminology

- **Block Diagrams are widely used in engineering and science and exist in many different forms.**

- **Reliability Block Diagram (RBD)**
  - **Describes the interrelation between the components to define the system**
  - **Graphical representation of the system components and how they are reliability-wise related (connected)**
  - **RBD may differ from how the components are physically connected**
  - **After defining properties of each block in a system, the blocks can be connected in a reliability-wise manner to create a RBD for the system**

Example Reliability Block Diagram

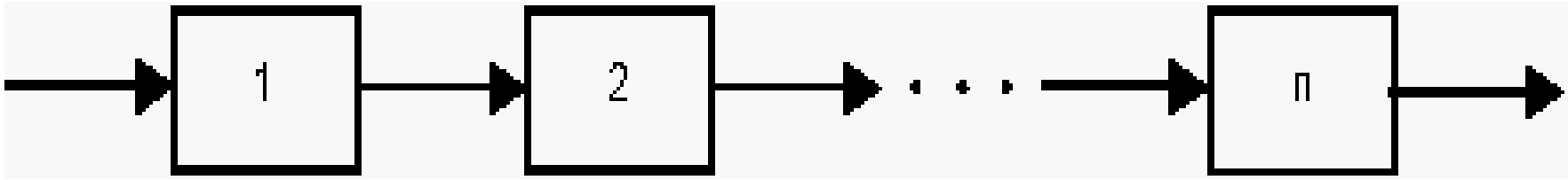
RBD of a simplified computer system with a redundant fan configuration



System Reliability Block Diagram

- **The System Reliability Function**
  - **The RBD represents the system's functioning state (*i.e.* success or failure) in terms of the functioning states of its components**
  - **The RBD demonstrates the effect of the success or failure of a component on the success or failure of the system**
    - **If all components in a system must succeed for the system to succeed, the components are arranged reliability-wise in *series***
    - **If one of two components must succeed in order for the system to succeed, those two components are arranged reliability-wise in parallel**
  - **The reliability-wise arrangement of components is directly related to the derived mathematical description of the system**
  - **The system's reliability function uses probabilistic methods for defining the system reliability from the component reliabilities**
  - **System reliability is often described as a function of time**

Series Configuration



- **A failure of any component results in failure for the entire system**
- **When considering a system at the subsystem level, subsystems are often arranged reliability-wise in a series configuration**
  - **Example: a PC may consist of four basic subsystems: the motherboard, hard drive, power supply and the processor**
  - **A failure to any of these subsystems will cause a system failure**
  - **All units in a series system must succeed for system to succeed**

Series Configuration System Reliability

- **The reliability of the system is the probability that unit 1 succeeds and unit 2 succeeds *and* all of the other units in the system succeed**
- **All *n* units must succeed for the system to succeed**

The reliability of the system is then given by:

$$R_s = P(X_1 \cap X_2 \cap \ldots \cap X_n)$$
$$= P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_1X_2)\ldots P(X_n \mid X_1X_2\ldots X_{n-1})$$

In the case of independent components, this becomes:

$$R_s = P(X_1)P(X_2)\ldots P(X_n)$$

Or:

$$R_s = \prod_{i=1}^{n} P(X_i)$$

Series System Reliability Example

- **Three subsystems are reliability-wise in series & make up a system**
  - **Subsystem 1 has a reliability of 99.5% for a 100 hour mission**
  - **Subsystem 2 has a reliability of 98.7% for a 100 hour mission**
  - **Subsystem 3 has a reliability of 97.3% for a 100 hour mission**
- **What is the overall reliability of the system for a 100 hour mission?**

- **Solution to the RBD and Analytical System Reliability Example**
  - **Since reliabilities of the subsystems are specified for 100 hours, the reliability of the system for a 100 hour mission is simply:**

$$R_S = R_1 \cdot R_2 \cdot R_3$$

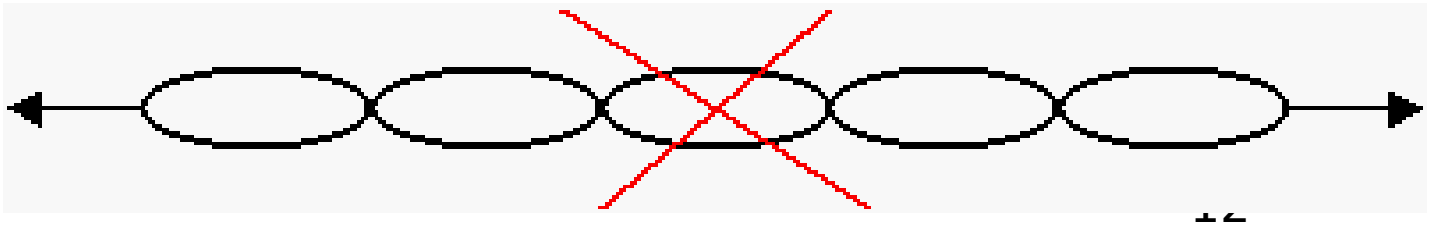
$$R_S = 0.9950 \cdot 0.9870 \cdot 0.9730$$

$$R_S = 0.955549245$$

$$R_S = 95.55\%$$

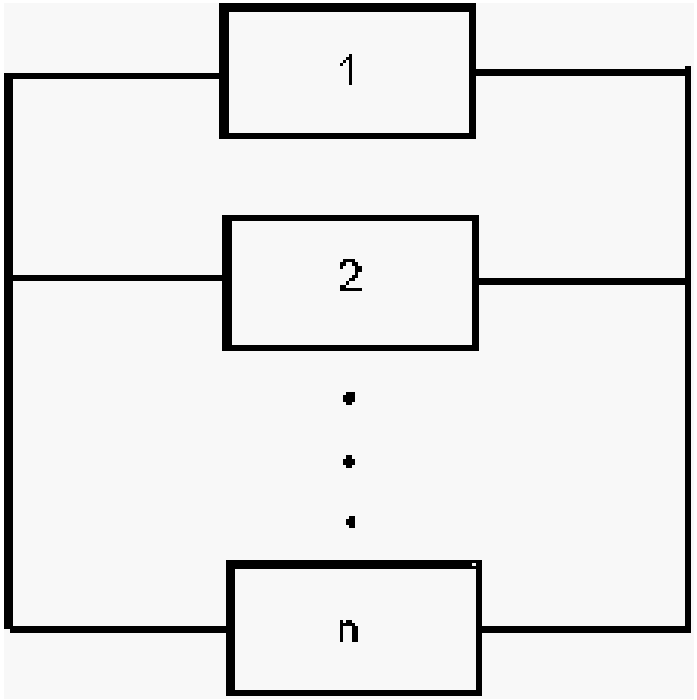
Basic System Reliability

- **Effect of Component Reliability in a Series System**
  - **In a series configuration, the component with the smallest reliability has the biggest effect on the system's reliability**
  - **Saying: A chain is only as strong as its weakest link**
  - **Good example of the effect of a component in a series system**
    - **In a chain, all the rings are in series and if any of the rings break, the system fails**
    - **The weakest link in the chain is the one that will break first**
    - **The weakest link dictates the strength of the chain in the same way that the weakest component/subsystem dictates the reliability of a series system**
  - **As a result, the reliability of a series system is always less than the reliability of the least reliable component.**



Redundant Configuration

- **Simple Parallel Systems**



Redundant System Configuration

- **In a simple parallel system, at least one of the units must succeed for the system to succeed**
- **Units in parallel are also referred to as *redundant* units**
- **Redundancy is a very important aspect of system design & reliability because adding redundancy is one of several methods to improve system reliability**
- **Redundancy is widely used in the aerospace industry and generally used in mission critical systems**

Parallel Configuration System Reliability

- **The probability of failure, or unreliability, for a system with *n* statistically independent parallel components is the probability that unit 1 fails *and* unit 2 fails *and* all of the other units in the system fail**
- **In a parallel system, all *n* units must fail for the system to fail**
- **If unit 1 succeeds or unit 2 succeeds or any of the *n* units succeeds, then the system succeeds**

The unreliability of the system is then given by:

$$Q_s = P(X_1 \cap X_2 \cap \ldots \cap X_n)$$
$$= P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_1X_2)\ldots P(X_n \mid X_1X_2\ldots X_{n-1})$$

Redundant System Unreliability

In the case of independent components:

$$Q_s = P(X_1)P(X_2)\ldots P(X_n) \quad \textbf{Or} \quad Q_s = \prod_{i=1}^{n} P(X_i)$$

Or, in terms of component unreliability:

$$Q_s = \prod_{i=1}^{n} Q_i$$

Redundant System Reliability

- **With the series system, the system reliability is the product of the component reliabilities**
- **With the parallel system, the overall system unreliability is the product of the component unreliabilities**

The reliability of the parallel system is then given by:

$$
\begin{aligned}
R_s &= 1 - Q_S = 1 - (Q_1 \cdot Q_2 \cdot \ldots \cdot Q_n) \\
&= 1 - [(1 - R_1) \cdot (1 - R_2) \cdot \ldots \cdot (1 - R_n)] \\
&= 1 - \prod_{i=1}^{n} (1 - R_i)
\end{aligned}
$$

Redundant System Reqt. Example

- **What is the MTBOMF of each system when it is required to have 91% probability that 1 of 2 systems operate failure free over a 72 hour mission pulse?**

$$R = 1 - Q \qquad Q_1 \cdot Q_2 = Q^2 = 0.09 \qquad \sqrt{Q^2} = \sqrt{0.09}$$

$$Q = 0.3 \qquad R = 1 - Q = 1 - 0.3 = 0.7 \text{ per system}$$

$$0.7 = e^{\frac{-72}{MTBOMF}} \qquad \ln(.7) = \ln\left[e^{\frac{-72}{MTBOMF}}\right]$$

$$-0.356675 = \frac{-72}{MTBOMF} \qquad 0.356675 \times MTBOMF = 72$$

$$MTBOMF = \frac{72}{0.356675} = 201.8 \text{ operating hours per mission failure}$$

Redundant System Reliability Example

- Three subsystems are reliability-wise in parallel & make up a system
  - Subsystem 1 has a reliability of 99.5% for a 100 hour mission
  - Subsystem 2 has a reliability of 98.7% for a 100 hour mission
  - Subsystem 3 has a reliability of 97.3% for a 100 hour mission
- What is the overall reliability of the system for a 100 hour mission?

- Solution to the RBD and Analytical System Reliability Example
  - Since reliabilities of the subsystems are specified for 100 hours, the reliability of the system for a 100 hour mission is simply:

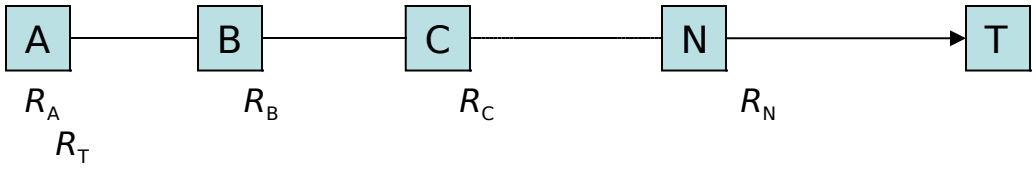
$$R_S = 1 - [(1 - 0.9950) \cdot (1 - 0.9870) \cdot (1 - 0.9730)]$$

$$R_S = 1 - 0.000001755$$

$$R_S = 0.999998245$$

$$R_S = 99.9998\%$$

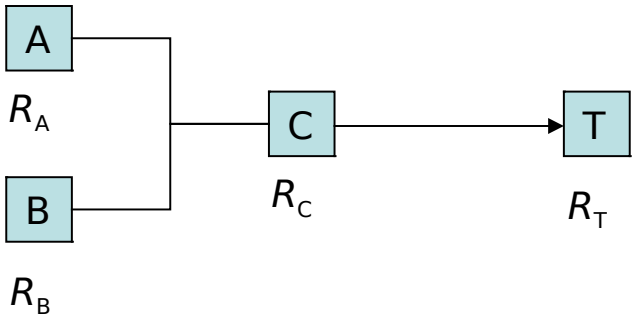
Series Reliability Block Diagram



All elements, (A,B,C,…,N) must work for equipment T to work. The reliability of T is:

$$R_T = R_A \bullet R_B \bullet R_C \bullet \ldots \bullet R_N \prod_{i=A}^{N} R_i$$
$$=$$

Block Diagrams with Parallel Reliability and Series Reliability



At least one of the elements (A,B) and element C must work for equipment T to work. The reliability of T is:

$$R_T = \left[1 - (1 - R_A)(1 - R_B)\right] x R_C$$
$$= \left[1 - (1 - R_A - R_B + R_A R_B)\right] x R_C$$
$$= \left[1 - 1 + R_A + R_B - R_A R_B\right] x R_C$$
$$= (R_A + R_B - R_A R_B) x R_C$$

Non-Repairable Systems

- **Non-repairable systems do not get repaired when they fail**
  - **Specifically, components of the system are not removed or replaced when the system fails because it does not make economic sense to repair the system**
  - **Repairing a four-year-old microwave oven is economically unreasonable when the repair costs approximately as much as purchasing a new unit**

Repairable Systems

- **Repairable systems get repaired when they fail**
  - **Repairs are done by replacing the failed components in system**
  - **Example: An automobile is a repairable system when rendered inoperative by a component or subsystem failure by typically removing & replacing the failed components rather than purchasing a new automobile**
  - **Failure distributions and repair distributions apply to repairable systems**
    - **A failure distribution describes the time it takes for a component to fail**
    - **A repair distribution describes the time it takes to repair a component (time-to-repair instead of time-to-failure)**
  - **For repairable systems, the failure distribution itself is not a sufficient measure of system performance because it does not account for the repair distribution**
  - **A performance criterion called availability is calculated to account for both the failure and repair distributions**

System Maintainability/Maintenance

- **Deals with repairable system maintenance**
- **System Maintainability involves the time it takes to restore a system to a specified condition when maintenance is performed by personnel having specified skills using prescribed procedures and resources**
- **In general, maintenance is defined as any action that restores failed units to an operational condition or retains non-failed units in an operational state**
- **Maintenance plays a vital role in the life of a system affecting the system's overall reliability, availability, downtime, cost of operation, etc.**
- **Types of system maintenance actions: corrective maintenance, preventive maintenance & inspections**

Corrective Maintenance

- **Actions taken to restore a failed system to operational status**
- **Usually involves replacing or repairing the component that is responsible for the failure of the overall system**
- **Corrective maintenance is performed at unpredictable intervals because a component's failure time is not known *a priori***
- **The objective of corrective maintenance is to restore the system to satisfactory operation within the shortest possible time**

Corrective Maintenance Steps

- **Diagnosis of the problem**
  - **Maintenance technician takes time to locate the failed parts or otherwise satisfactorily assess the cause of the system failure**
- **Repair and/or replacement of faulty component**
  - **Action is taken to address the cause, usually by replacing or repairing the components that caused the system to fail**
- **Verification of the repair action**
  - **Once components have been repaired or replaced, the maintenance technician must verify that the system is again successfully operating**

Preventive Maintenance

- **The practice of replacing components or subsystems before they fail to promote continuous system operation**
- **The preventive maintenance schedule is based on:**
  - **Observation of past system behavior**
  - **Component wear-out mechanisms**
  - **Knowledge of components vital to continued system operation**
- **Cost is always a factor in the scheduling of preventive maintenance**
  - **Reliability may be a factor, but cost is a more general term because reliability & risk can be expressed in terms of cost**
  - **In many circumstances, it may be financially better to replace parts or components that have not failed at predetermined intervals rather than wait for a system failure that may result in a costly disruption in operations**

Inspections

- **Used to uncover hidden failures (also called dormant failures)**
- **In general, no maintenance action is performed on the component during an inspection unless the component is found failed causing a corrective maintenance action to be initiated**
- **Sometimes there may be a partial restoration of the inspected item performed during an inspection**
  - **For example, when checking the motor oil in a car between scheduled oil changes, one might occasionally add some oil in order to keep it at a constant level**

Maintenance Downtime

- **There is time associated with each maintenance action, *i.e.* amount of time it takes to complete the action**
- **This time is referred to as *downtime* & defined as the length of time an item is not operational**
- **There are a number of different factors that can affect the length of downtime**
  - **Physical characteristics of the system**
  - **Repair crew availability**
  - **Spare part availability & other ILS factors**
  - **Human factors & Environmental factors**
- **There are two Downtime categories for these factors: Waiting Downtime & Active Downtime**

Maintenance Downtime

- **Waiting Downtime**
  - **The time during which the equipment is inoperable, but not yet undergoing repair**
  - **For example, the time it takes for replacement parts to be shipped, administrative processing time, etc.**

- **Active Downtime**
  - **The time during which the equipment is inoperable and actually undergoing repair**
  - **The active downtime is the time it takes repair personnel to perform a repair or replacement**
  - **The length of the active downtime is greatly dependent on human factors and the design of the equipment**
  - **For example, the ease of accessibility of components in a system has a direct effect on the active downtime**

System Maintainability

- **The time it takes to repair/restore a specific item is a random variable implying an underlying probabilistic distribution**

- **Distributions describing the time-to-repair are *repair or downtime distributions*, distinguishing them from *failure distributions***

- **Methods to quantify these distributions are similar, but differ in how employed, *i.e.* the events they describe and metrics utilized**
  - **In failure distributions, *unreliability* provides the probability the event (failure) will occur by that time, while *reliability* provides the probability the event (failure) will not occur**
  - **In downtime distributions, the times-to-repair data becomes the probability of the event (repairing the component) occurring**

- **The probability of repairing the component by a given time, *t*, is also called the component's *maintainability***

System Maintainability

- **Maintainability is sometimes defined as a probability of performing a successful repair action within a given time**
- **Measures the ease & speed with which a system can be restored to operational status after a failure occurs**
- **For example, a component with a 90% maintainability in one hour has a 90% probability the component will be repaired in one hour**
- **Maintainability M(t) for a system with the repair times distributed exponentially is given by:**

$$M(t) = 1 - e^{-\mu t}$$ **where**

μ = repair rate $\frac{1}{\mu}$ **= Mean Time To Repair (MTTR)**

Maintainability/Time to Repair Terms

- **Mean Corrective Maintenance Time for Operational Mission Failure Repairs (MCMTOMF) is based on the average time to repair operational mission failures**
- **Mean Corrective Maintenance Time (MCMT) is based on the average corrective time to all failures**
- **Maximum (e.g. 90 percentile time) Corrective Maintenance Time (MaxCMT) for all incidents may be applied to maintainability testing**
- **Maintenance Ratio (MR) is a full maintenance burden requirement expressed in terms of the Mean Maintenance Man-Hours per Operating Hour, Mile, etc. The cumulative number of maintenance man-hours during a given period divided by the cumulative number of operating hours**

Availability

- **Considers both reliability (probability the item will not fail) and maintainability (probability the item is successfully restored after failure)**
- **Reliability, Availability, and Maintainability (RAM) are always associated with time**
- **Availability is the probability that the system/component is operational at a given time, *t* (*i.e.* has not failed or it has been restored after failure)**
- **May be defined as the probability an item is operable & can be committed at the start of a mission when the mission is called for at any unknown (random) point in time. Example: For a lamp with a 99.9% availability, there will be one time out of a thousand that someone needs to use the lamp and finds it is not operating**

RAM Relationships

- **Availability alone tells us nothing about how many times the lamp has been replaced**
- **Reliability and Maintainability metrics are still important. The table illustrates RAM relationships**

Reliability	Maintainability	Availability
▬ Constant	↓ Decreases	↓ Decreases
▬ Constant	↑ Increases	↑ Increases
↑ Increases	▬ Constant	↑ Increases
↓ Decreases	▬ Constant	↓ Decreases

Inherent Availability

- **The steady state availability when considering only the corrective downtime of the system**
  - **For a single component, this can be computed by:**

$$A_I = \frac{MTTF}{MTTF + MTTR}$$

- For a system, the Mean Time Between Failures, or MTBF, is used to compute inherent availability:

$$A_I = \frac{MTBF}{MTBF + MTTR}$$

Achieved Availability

- **Achieved Availability is similar to Inherent Availability except Preventive Maintenance (PM) is also included**
- **The steady state availability when considering the corrective and preventive downtime of the system**
- **Computed by looking at the Mean Time Between Maintenance actions, *MTBM* and the Mean Maintenance Downtime:**

$$A_A = \frac{MTBM}{MTBM + \overline{M}}$$

Operational Availability

- **Operational Availability is the percentage of calendar time to which one can expect a system to work properly when it is required**
- **Expression of User Need rather than just Design Need**
- **Operational Availability is the ratio of the system Uptime and Total time. Mathematically, it is:**

$$A_o = \frac{Uptime}{Uptime + Downtime}$$

- **Includes all experienced sources of downtime, such as administrative downtime and logistic downtime to restore the system**

Basic System Availability

- **Previous availability definitions can be *a priori* estimations based on models of the system failure and downtime distributions**
- **Inherent Availability and Achieved Availability are controlled by the system designer/manufacturer**
- **Operational Availability is not solely controlled by the manufacturer due to variations in location, resources and logistics factors under the province of the end user of the product**
- **When recorded, an Operational Readiness Rate is the Operational Availability that the customer actually experiences. It is the *a posteriori* availability based on actual events that happened to the system**

Ao / Operational Readiness Example

- **A diesel power generator is supplying electricity at a research site in Antarctica & personnel are not satisfied with the generator**
- **In the past six months, they estimate being without electricity due to generator failure for an accumulated time of 1.5 months**
- **Therefore, the operational availability of the diesel generator experienced by personnel of the station is:**

$$A_o = \frac{4.5 \text{ months}}{6 \text{ months}} = 0.75$$

Redundant Configurations

- **Hot Standby Redundancy**
  - **Operates all systems or subassemblies simultaneously**
  - **Accrues more failures by operating all items**
  - **Switchover time to the redundant item is near instantaneous**
  - **Uses the Binomial Distribution to determine the Operational Availability (Ao) of the redundant configuration**

- **Cold Standby Redundancy**
  - **Redundant systems or subassemblies are treated like spares stored in the system configuration**
  - **Accrues less failures by operating only the items needed**
  - **Switchover time to the redundant item is needed**
  - **Uses the Poisson Distribution to determine the Ao of the redundant configuration**

Binomial Distribution

R out of N of the Same System Need To Be Up:

$$A_o config = A_o^N + N \times A_o^{N-1} \times (1 - A_o) + \frac{N(N-1)}{2} A_o^{N-2} \times (1 - A_o)^2 + \ldots + \frac{N!}{(N-R)!R!} \times A_o^{N-R} \times (1 - A_o)^R$$

Series configuration where R=N as all common items need to be up:

$A_o config = A_o^N$ **because only the first Binomial term is used**

Redundant Config where R=1 as only 1 of the items needs to be up:

Note: All terms of a Binomial Distribution sum up to 1

$A_o config = 1 - (1 - A_o)^N$ **because all but the last Binomial term is used**